UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2008**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

8114763367

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

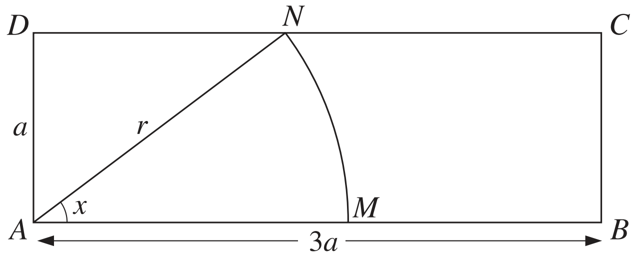
 [Turn over

1 Solve the inequality $|x-2| > 3|2x+1|$. [4]

2 Solve, correct to 3 significant figures, the equation

$$e^x + e^{2x} = e^{3x}.$$ [5]

3



In the diagram, $ABCD$ is a rectangle with $AB = 3a$ and $AD = a$. A circular arc, with centre A and radius r, joins points M and N on AB and CD respectively. The angle MAN is x radians. The perimeter of the sector AMN is equal to half the perimeter of the rectangle.

(i) Show that x satisfies the equation

$$\sin x = \tfrac{1}{4}(2+x).$$ [3]

(ii) This equation has only one root in the interval $0 < x < \frac{1}{2}\pi$. Use the iterative formula

$$x_{n+1} = \sin^{-1}\left(\frac{2+x_n}{4}\right),$$

with initial value $x_1 = 0.8$, to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 **(i)** Show that the equation $\tan(30° + \theta) = 2\tan(60° - \theta)$ can be written in the form

$$\tan^2\theta + (6\sqrt{3})\tan\theta - 5 = 0.$$ [4]

(ii) Hence, or otherwise, solve the equation

$$\tan(30° + \theta) = 2\tan(60° - \theta),$$

for $0° \leqslant \theta \leqslant 180°$. [3]

5 The variable complex number z is given by

$$z = 2\cos\theta + \mathrm{i}(1 - 2\sin\theta),$$

where θ takes all values in the interval $-\pi < \theta \leqslant \pi$.

(i) Show that $|z - \mathrm{i}| = 2$, for all values of θ. Hence sketch, in an Argand diagram, the locus of the point representing z. [3]

(ii) Prove that the real part of $\dfrac{1}{z + 2 - \mathrm{i}}$ is constant for $-\pi < \theta < \pi$. [4]

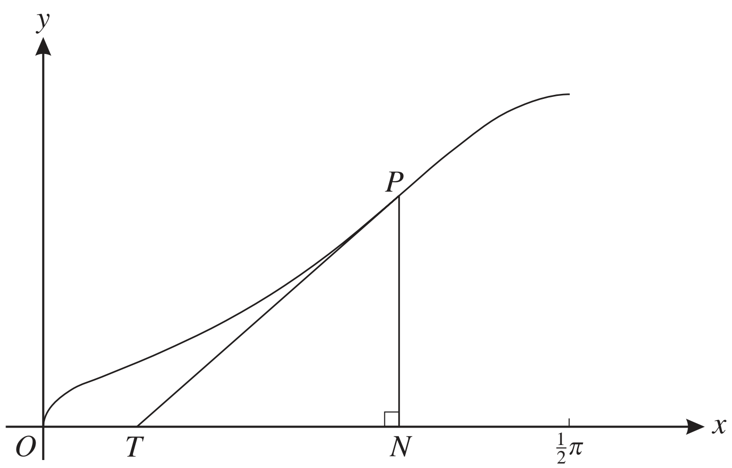
6 The equation of a curve is $xy(x + y) = 2a^3$, where a is a non-zero constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis, and find the coordinates of this point. [8]

7 Let $\mathrm{f}(x) \equiv \dfrac{x^2 + 3x + 3}{(x + 1)(x + 3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_0^3 \mathrm{f}(x)\,\mathrm{d}x = 3 - \tfrac{1}{2}\ln 2$. [4]

8



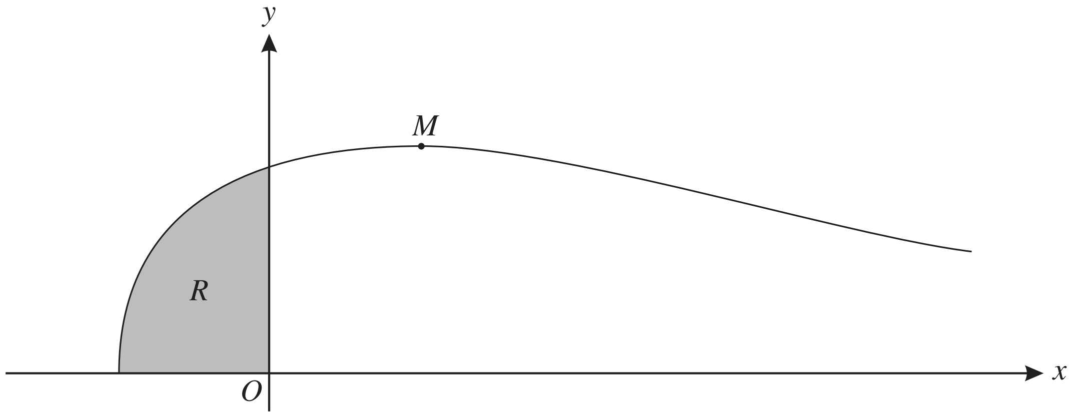
In the diagram the tangent to a curve at a general point P with coordinates (x, y) meets the x-axis at T. The point N on the x-axis is such that PN is perpendicular to the x-axis. The curve is such that, for all values of x in the interval $0 < x < \frac{1}{2}\pi$, the area of triangle PTN is equal to $\tan x$, where x is in radians.

(i) Using the fact that the gradient of the curve at P is $\dfrac{PN}{TN}$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2}y^2 \cot x.$$ [3]

(ii) Given that $y = 2$ when $x = \frac{1}{6}\pi$, solve this differential equation to find the equation of the curve, expressing y in terms of x. [6]

9



The diagram shows the curve $y = e^{-\frac{1}{2}x}\sqrt{(1 + 2x)}$ and its maximum point M. The shaded region between the curve and the axes is denoted by R.

(i) Find the x-coordinate of M. [4]

(ii) Find by integration the volume of the solid obtained when R is rotated completely about the x-axis. Give your answer in terms of π and e. [6]

10 The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation

$$\mathbf{r} = (1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}.$$

(i) Show that l does not intersect the line passing through A and B. [4]

(ii) The point P lies on l and is such that angle PAB is equal to $60°$. Given that the position vector of P is $(1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}$, show that $3t^2 + 7t + 2 = 0$. Hence find the only possible position vector of P. [6]